Elements of Proof: Joint Stochastic Geometry and Mean Field Game Optimization for Energy-Efficient Proactive Scheduling in Ultra Dense Networks

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This paper contains proofs related to the following paper:

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I. Proof of: $h_{j,i}$ can be modeled as a SDE.

Proposition I.1. *Let us assume that*

1) *the BS positions $Z_i^b = (X_j^b, Y_j^b)$ are fixed wrt. time and that the users positions can be modeled with an SDE, as:*

$$\begin{aligned}
&Z_i^u(t+1) = Z_i^u(t) + \alpha_i^u(t, Z_i^u(t))\Delta_t + \sigma_i^u(t)W_i^u(t) \\
&\Leftrightarrow \begin{cases} X_i^u(t+1) = X_i^u(t) + \alpha_{i,X}^u(t, X_i^u(t))\Delta_t + \sigma_i^{u,X}(t)W_i^{u,X}(t) \\ Y_i^u(t+1) = Y_i^u(t) + \alpha_{i,Y}^u(t, Y_i^u(t))\Delta_t + \sigma_i^{u,Y}(t)W_i^{u,Y}(t) \end{cases} \\
&W_i^{u,X}(t), W_i^{u,Y}(t) \rightarrow \mathcal{N}(0, \Delta_t) \\
&\text{With initial position } Z_i^u(T_i^s) \text{ known.}
\end{aligned} \tag{1}$$

2) *the distance between a BS j and a user i at time t , denoted $d_{j,i}(t)$ follows from the Euclidian norm:*

$$d_{j,i}(t) = \sqrt{(X_j^b - X_i^u(t))^2 + (Y_j^b - Y_i^u(t))^2} \tag{2}$$

3) *the channel gain $h_{j,i}(t)$ between BS j and user i at time t, writes as:*

$$h_{j,i}(t) = l_{j,i}(t)|g_{j,i}(t)|^2 \tag{3}$$

with $l_{j,i}(t)$, the path loss based on the distance $d_{j,i}(t)$:

$$l_{j,i}(t) = \min\left(1, \left(\frac{1}{d_{j,i}(t)}\right)^{\beta_{j,i}(t)}\right) \tag{4}$$

and $g_{j,i}(t)$ *a fading, following a special kind of SDE known as Ornstein-Uhlenbeck process [2] [3], which writes:*

$$g_{j,i}(t+1) = g_{j,i}(t) + \frac{(\alpha^g_{j,i} - g_{j,i}(t))\Delta_t}{2} + \sigma^g_{j,i}(t)W^g_{j,i}(t) \tag{5}$$

where $W^g_{j,i}(t)$ *is a Wiener process following* $\mathcal{N}(0,\Delta_t)$*, and the values of* $\alpha^g_{j,i}$ *and* $\sigma^g_{j,i}(t)$ *allows to capture different channel statistics, including slow and fast fading [4].*

Proposition: $h_{j,i}$ ***also follows a SDE.***

Proof. The core idea here is to first express $d_{j,i}(t)$ as an SDE using Itô's Lemma. The distance function given in Eq. (2) is twice differentiable and depends on the user's position $Z^u_i(t)$, which follows the SDE given in Eq. (1). Using Itô's lemma, we can then define $\alpha^d_{j,i}$ and $\sigma^d_{j,i}$, such that $d_{j,i}(t)$ follows an SDE, detailed hereafter:

$$d_{j,i}(t+1) = d_{j,i}(t) + \alpha^d_{j,i}(t, d_{j,i}(t))\Delta_t + \sigma^d_{j,i}(t)\mathcal{B}_{j,i}(t) \tag{6}$$

We can then repeat the process with the path loss function $l_{j,i}(t)$ (which is again twice differentiable), detailed in Eq. (4), and the newly obtained SDE (6). Using Itô's lemma a second time, we can then define $\alpha^l_{j,i}$ and $\sigma^l_{j,i}$, such that $l_{j,i}(t)$ follows an SDE, detailed hereafter:

$$l_{j,i}(t+1) = l_{j,i}(t) + \alpha^l_{j,i}(t, l_{j,i}(t))\Delta_t + \sigma^l_{j,i}(t)\mathcal{B}_{j,i}(t) \tag{7}$$

The final step consists of using Itô's product rule, to obtain $h_{j,i}(t)$, defined in Eq. (3), as the product of both SDEs (5) and (7). As a consequence, we can define $\alpha^h_{j,i}$ and $\sigma^h_{j,i}$, so that $h_{j,i}$ writes as an SDE:

$$h_{j,i}(t+1) = h_{j,i}(t) + \alpha^h_{j,i}(t, h_{j,i}(t))\Delta_t + \sigma^h_{j,i}(t)\mathcal{B}_{j,i}(t) \tag{8}$$

□

Note: we will later update the exact calculations for $\alpha^d_{j,i}$**,** $\sigma^d_{j,i}$**,** $\alpha^l_{j,i}$**,** $\sigma^l_{j,i}$**,** $\alpha^h_{j,i}$**, and** $\sigma^h_{j,i}$**.**

II. Proof of: $\tilde{h}^a_i(t)$ can be modeled as a SDE.

Proposition II.1. *Let us assume the channels follow a SDE:*

$$h_{j,i}(t+1) = h_{j,i}(t) + \alpha^h_{j,i}(t, h_{j,i}(t))\Delta_t + \sigma^h_{j,i}(t)\mathcal{B}_{j,i}(t) \tag{9}$$

We denote the assigned transmission channel $\tilde{h}^a_i(t) = h_{A_i(t),i}(t)$*, where* $A_i(t)$ *is the BS index corresponding to the BS assigned to user* i *at time* t*. Also, we assume* $\mathcal{H}(1) = \left(h_{j,i}(1)\right)_{i,j}$ *is known.*

Proposition: $\tilde{h}^a_i$ ***can be modeled as a SDE:***

$$\tilde{h}^a_i(t+1) = \tilde{h}^a_i(t) + \tilde{\alpha}^a_i(t)\Delta_t + \tilde{\sigma}^a_i(t)\tilde{\mathcal{B}}^a_i(t) \tag{10}$$

This notation is the discrete time equivalent notation of a continuous time SDE.

Proof. Two cases need to be investigated here:

- The non-handover case: $A_i(t+1) = A_i(t)$

- The handover case: $A_i(t+1) \neq A_i(t)$

Let us start with the non-handover case. Since $A_i(t+1) = A_i(t)$, we can immediately reuse SDE (9) and define $\tilde{\alpha}_i^a(t, h_i^a(t) = \alpha^h_{A_i(t),i}(t, h_{A_i(t),i}(t))$ and $\tilde{\sigma}_i^a(t) = \sigma^h_{A_i(t),i}(t)$.

It get more complicated when a handover happens. Let us denote, $j' = A_i(t+1)$ and $j = A_i(t)$ and begin from the SDE:

$$h^a_{j',i}(t+1) = h_{j',i}(t) + \alpha^h_{j',i}(t)\Delta_t + \sigma_{j',i}(t)\mathcal{B}_{j',i}(t) \tag{11}$$

We first have to force the apparition of the $\tilde{h}_i^a(t)$ term:

$$\tilde{h}_i^a(t+1) = \tilde{h}_i^a(t) + \left(h_{j',i}(t) - h_{j,i}(t)\right) + \alpha^h_{j',i}(t)\Delta_t + \sigma_{j',i}(t)\mathcal{B}_{j',i}(t) \tag{12}$$

Realize that $h_{j',i}(t)$ and $h_{j,i}(t)$ can be defined using the SDEs:

$$\begin{aligned} h_{j',i}(t) &= h_{j',i}(1) + \sum_{t'=1}^{t-1} \alpha^h_{j',i}(t')\Delta_t + \sum_{t'=1}^{t-1} \sigma^h_{j',i}(t')\mathcal{B}_{j',i}(t) \\ h_{j,i}(t) &= h_{j,i}(1) + \sum_{t'=1}^{t-1} \alpha^h_{j,i}(t')\Delta_t + \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\mathcal{B}_{j,i}(t) \end{aligned} \tag{13}$$

Substituting in the previous equation leads to

$$\begin{aligned} h_i^a(t+1) &= h_i^a(t) + \left(h_{j',i}(1) - hj,i(1)\right) + \sum_{t'=1}^{t-1} \alpha^h_{j',i}(t')\Delta_t - \sum_{t'=1}^{t-1} \alpha^h_{j,i}(t')\Delta_t + \alpha^h_{j',i}(t)\Delta_t \\ &\quad + \sum_{t'=1}^{t-1} \sigma^h_{j',i}(t')\mathcal{B}_{j',i}(t) - \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\mathcal{B}_{j,i}(t) + \sigma_{j',i}(t)\mathcal{B}_{j',i}(t) \\ &= h_i^a(t) + \underbrace{\left(h_{j',i}(1) - hj,i(1)\right) + \sum_{t'=1}^{t} \alpha^h_{j',i}(t')\Delta_t - \sum_{t'=1}^{t-1} \alpha^h_{j,i}(t')\Delta_t}_{\text{(A)}} + \underbrace{\sum_{t'=1}^{t} \sigma^h_{j',i}(t')\mathcal{B}_{j',i}(t) - \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\mathcal{B}_{j,i}(t)}_{\text{(B)}} \end{aligned} \tag{14}$$

Here, part(A) will play the role of $\tilde{\alpha}_i^a(t)\Delta_t$ and thus:

$$\tilde{\alpha}_i^a(t) = \frac{h_{j',i}(1) - hj,i(1)}{Delta_t} + \sum_{t'=1}^{t} \alpha^h_{j',i}(t') - \sum_{t'=1}^{t-1} \alpha^h_{j,i}(t') \tag{15}$$

Note that when no handover occurs, i.e. $j' = j$, the term simplifies as $\tilde{\alpha}_i^a(t) = \alpha^h_{j,i}(t)$. Part (B) is playing the role of the stochastic part. Since the $\mathcal{B}_.$ are Wiener processes, we can write:

$$\begin{aligned} \text{(B)} &= \sum_{t'=1}^{t} \sigma^h_{j',i}(t')\mathcal{B}_{j',i}(t) - \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\mathcal{B}_{j,i}(t) \\ &= \sum_{t'=1}^{t} \sigma^h_{j',i}(t')\mathcal{B}_{j',i}(t) + \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\mathcal{B}_{j,i}(t) \\ &= \left(\sum_{t'=1}^{t} \sigma^h_{j',i}(t') + \sum_{t'=1}^{t-1} \sigma^h_{j,i}(t')\right)\mathcal{B}_i^a(t) \end{aligned} \tag{16}$$

With $\tilde{\mathcal{B}}_i^a(t)$ a unique Wiener process, following from the summation property of the $(2t-1)$ Wiener processes.

To conclude, we can write $\tilde{h}_i^a$ as a SDE:

$$\tilde{h}_i^a(t+1) = \tilde{h}_i^a(t) + \tilde{\alpha}_i^a(t)\Delta_t + \tilde{\sigma}_i^a(t)\tilde{\mathcal{B}}_i^a(t) \tag{17}$$

With

$$\tilde{\alpha}_i^a(t) = \frac{h_{a_i(t+1),i}(1) - hA_i(t),i(1)}{Delta_t} + \sum_{t'=1}^{t} \alpha^h_{A_i(t+1),i}(t') - \sum_{t'=1}^{t-1} \alpha^h_{A_i(t),i}(t') \tag{18}$$

And

$$\tilde{\sigma}_i^a(t) = \begin{cases} \sum_{t'=1}^{t} \sigma_{j',i}^h(t') + \sum_{t'=1}^{t-1} \sigma_{j,i}^h(t') & \text{if a handover occurs,} \\ \sigma_{A_i(t),i}(t) & \text{otherwise.} \end{cases} \tag{19}$$

Finally, $\tilde{\mathcal{B}}_i^a(t)$ refers to a Wiener process. □

III. Proof of: $h_i^a(k)$ can be modeled as a SDE.

Proposition III.1. *Let us assume the channels follow a SDE:*

$$\tilde{h}_i^a(t+1) = \tilde{h}_i^a(t) + \tilde{\alpha}_i^a(t, h_i^a(t))\Delta_t + \tilde{\sigma}_i^a(t)\tilde{\mathcal{B}}_i^a(t) \tag{20}$$

We denote the assigned transmission channel $h_i^a(k) = h_{A_i(\pi_i^t(k)),i}(\pi_i^t(k))$, where $A_i(t)$ is the BS index corresponding to the BS assigned to user i at time t. Here, $k \in K_i$ and $\pi_i^t(k)$ refers to the time slot index of $\pi_i(k), k \in K_i$.

Proposition: h_i^a can be modeled as a SDE:

$$h_i^a(k-1) = h_i^a(k) + \alpha_i^a(k)\Delta_t + \sigma_i^a(k)\mathcal{B}_i^a(k) \tag{21}$$

Proof. The key here is to realize the channel changes if and only if $\pi_i^w(k)$ corresponds to the last bandwidth element assigned to user i during time slot $\pi_i^t(k)$, i.e. if and only if $k \in K_i^l$. Otherwise, the channel remains unvariant.

Following from this we simply reuse the previous result and obtain:

$$\tilde{\alpha}_i^a(t) = \begin{cases} \frac{h_{a_i(t+1),i}(1) - hA_i(t),i(1)}{\Delta_t} + \sum_{t'=1}^{t} \alpha_{A_i(t+1),i}^h(t') - \sum_{t'=1}^{t-1} \alpha_{A_i(t),i}^h(t') & \text{if } k \in K_i^l, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

Also, we have:

$$\sigma_i^a(k) = \begin{cases} \sigma_{A_i(\pi_i^t(k-1)),i}^h(\pi_i^t(k)) & \text{if } k \in K_i^l \text{ and no handover occurs.,} \\ \sum_{k'=k}^{|K_i|} \sigma_{A_i(\pi_i^t(k-1)),i}^h(\pi_i^t(k')) + \sum_{k'=k+1}^{|K_i|} \sigma_{A_i(\pi_i^t(k)),i}^h(\pi_i^t(k')) & \text{if } k \in K_i^l \text{ and handover occurs.} \\ 0 & \text{else.} \end{cases} \tag{23}$$

Finally, we just pose $\mathcal{B}_i^a(k) = \tilde{\mathcal{B}}_i^a(\pi_i^t(k))$. □

IV. HJB equation in the DSG.

Proposition IV.1. *Let us assume the optimization follows from:*

$$p_i(k, X_i(k))^* = \underset{p_i(k, X_i(k))}{\operatorname{argmin}} \left[p_i(k, X_i(k))\Delta_t + V_i(k-1, X_i(k-1))\right] \tag{24}$$

Where, $X_i(k) = [h_i^a(k), Q_i(k)]$. And, $V_i(k-1, X_i(k-1))$ is the Bellman function [5], which consists of the expected energy consumption on the remaining $k-1$ REs, if user i arrives at RE $k-1$ in state $X_i(k-1)$, i.e.:

$$V_i(k-1, X_i(k-1)) = \sum_{\substack{k' \in K_i \\ k' \leq k-1}} \left(\mathbb{E}_{X_i(k')}\left[p_i^*(k)\Delta_t\right]\right) + \mathbb{E}\left[\epsilon(Q_i(0))\right] \tag{25}$$

And ϵ is a penalty function, based on the final queue state $Q_i(0)$.

Proposition: The optimal trajectory of the Bellman function V_i^* follows from the HJB equation:

$$\begin{aligned} p_i(k', X_i(k')) + \min_{p_i(k',X_i(k'))} \Bigg[\frac{\partial V_i^*(k',X_i(k'))}{\partial k'} - \log_2\left(1 + \frac{p_i(k',X_i(k'))h_i^a(k')}{\sigma_n^2 + I_i(\pi_i(k'))}\right)\Delta_w \frac{\partial V_i^*(k',X_i(k'))}{\partial Q_i(k')} \\ + \alpha_i^a(k') \frac{\partial V_i^*(k',X_i(k'))}{\partial h_i^a(k')} + \frac{(\sigma_i^{b,a}(k'))^2}{2} \frac{\partial^2 V_i^*(k,X_i(k'))}{\partial h_i^a(k')^2} \Bigg] = 0 \end{aligned} \tag{26}$$

Proof. Equation (26) follows from the Taylor expansion of the term $V_i(k-1, X_i(k-1)$ in Equation (24) [2] [6]. □

V. Optimal power strategy equation in the DSG.

Proposition V.1. ***Proposition: If the optimal trajectory of the Bellman function V_i^* is known, then the optimal power strategy $p_i^*(t)$ follows:***

$$\forall k' \in K_i, p_i^*(k', X_i(k')) = \left[\frac{\partial V_i^*(k', X_i(k'))}{\partial Q_i(k')} \frac{\Delta_w}{\log(2)} - \frac{\sigma_n^2 + I_i(\pi_i(k'))}{h_i^a(k')} \right]^+ \tag{27}$$

***Where** $[x]^+ = \min(P^{max}, \max(0, x))$.*

Proof. The optimal power strategy $p_i^*(t)$ satisfies to the optimal min term in the HJB Equation (26) [2] [6], i.e.:

$$\begin{aligned} p_i^*(k', X_i(k')) &= \operatorname{argmin}_{p_i(k',X_i(k'))} \Bigg[\frac{\partial V_i^*(k',X_i(k'))}{\partial k'} - \log_2\left(1 + \frac{p_i(k',X_i(k'))h_i^a(k')}{\sigma_n^2 + I_i(\pi_i(k'))}\right)\Delta_w \frac{\partial V_i^*(k',X_i(k'))}{\partial Q_i(k')} \\ &\qquad + \alpha_i^a(k') \frac{\partial V_i^*(k',X_i(k'))}{\partial h_i^a(k')} + \frac{(\sigma_i^{b,a}(k'))^2}{2} \frac{\partial^2 V_i^*(k,X_i(k'))}{\partial h_i^a(k')^2} \Bigg] \\ &= \operatorname{argmin}_{p_i(k',X_i(k'))} \left[F(p_i(k', X_i(k')))\right] \end{aligned} \tag{28}$$

It follows from the derivation, that $p_i^*(k', X_i(k'))$ verifies:

$$\frac{\partial F(p_i^*(k', X_i(k')))}{\partial p_i(k', X_i(k'))} = 0. \tag{29}$$

Isolating $p_i^*(k', X_i(k'))$ in the previous equation leads to Equation (27). □

VI. HJB equation in the MFG.

The proof for the HJB equation in the MFG follows the same logic and hardly differs from the one detailed in Section IV.

VII. Optimal power strategy equation in the MFG.

The proof for the optimal power strategy equation in the MFG follows the same logic and hardly differs from the one detailed in Section V.

VIII. FPK EQUATION IN THE DSG.

Author's note: we are currently reworking this proof. It will be extensively detailed. It follows from our MFG definition, using theory from [2] [7].

IX. MEAN FIELD INTERFERENCE IN THE DSG.

Proposition IX.1. *Let us assume the interference term was originally expressed in the DSG, as*

$$I_i(t,w) = \sum_{\substack{i' \in \mathcal{N} \\ i' \neq i}} \delta\left(w \in \mathcal{W}_{i'}(t)\right) p_{i'}(t,w) h_{A_{i'}(t),i}(t) \tag{30}$$

Proposition: when the present resource is $k \in K_0$, the estimated mean field interference term for the typical user at RE $k' \in K_0, k > k'$can be written as:

$$I_0(\pi_0(k')) = |\Gamma(\pi_0(k'))| \mathbb{E}\left[h_0^{int}\right] \mathbb{E}\left[p_0^{int}\right] - \int\limits_{Y_0(k')} h_0^a(k')\tilde{p}(k', Y_0(k')) M_0^{Y_0(k)}(k', Y_0(k')) dY_0(k') \tag{31}$$

The convergence of this term can also be established.

Proof. Let us first consider the initial equation and change it slightly:

$$I_i(t,w) = \sum_{i' \in \mathcal{N}} \delta\left(w \in \mathcal{W}_{i'}(t)\right) p_{i'}(t,w) h_{A_{i'},i}(t) - \delta\left(w \in \mathcal{W}_{i}(t)\right) p_{i}(t,w) h_{A_{i}(t),i}(t) \tag{32}$$

In particular, it is true for our typical user $i = 0$.

$$I_0(t,w) = \sum_{i' \in \mathcal{N}} \delta\left(w \in \mathcal{W}_{i'}(t)\right) p_{i'}(t,w) h_{A_{i'},0}(t) - \delta\left(w \in \mathcal{W}_{0}(t)\right) p_{0}(t,w) h_{A_{0}(t),0}(t) \tag{33}$$

Let us denote $k, k' \in K_0$ and $k > k'$, the expected interference perceived by the typical user at time k' will then rewrite as:

$$\begin{aligned} I_0(\pi_0(k')) &= \mathbb{E}\left[\sum_{i' \in \mathcal{N}} \delta\left(\pi_0^w(k') \in \mathcal{W}_{i'}(\pi_0^t(k'))\right) p_{i'}(\pi_0(k')) h_{A_{i'}(\pi_0^t(k')),0}(\pi_0^t(k')) - p_0(\pi_0(k)) h_{A_0(\pi_0^t(k')),0}(\pi_0^t(k'))\right] \\ &= \underbrace{\mathbb{E}\left[\sum_{i' \in \mathcal{N}} \delta\left(\pi_0^w(k') \in \mathcal{W}_{i'}(\pi_0^t(k'))\right) p_{i'}(\pi_0(k')) h_{A_{i'}(\pi_0^t(k')),0}(\pi_0^t(k'))\right]}_{\text{(A)}} - \underbrace{\mathbb{E}\left[p_0(\pi_0(k)) h_{A_0(\pi_0^t(k')),0}(\pi_0^t(k'))\right]}_{\text{(B)}} \end{aligned} \tag{34}$$

Let us focus on part (B) first, reusing the definitions introduced in the FPK section, (B) rewrites as:

$$\text{(B)} = \int\limits_{Y_0(k') \in \mathcal{Y}} h_0^a(k')\tilde{p}(k', Y_0(k')) M_0^{Y_0(k)}(k', Y_0(k')) dY_0(k') \tag{35}$$

Regarding part (A), we must first, introduce the set of active user on resource $\pi_0(k')$, that we denoted $\Gamma(\pi_0(k'))$. Part (A) can thus rewrite as:

$$\text{(A)} = |\Gamma(\pi_0(k'))| \mathbb{E}\left[\frac{1}{|\Gamma(\pi_0(k'))|} \sum_{i' \in \mathcal{N}} \delta\left(\pi_0^w(k') \in \mathcal{W}_{i'}(\pi_0^t(k'))\right) p_{i'}(\pi_0(k')) h_{A_{i'}(\pi_0^t(k')),0}(\pi_0^t(k'))\right] \tag{36}$$

The expectation term here consist of the expectation of the averaged sum of the all the products of the power used by all active elements i' in set $\Gamma(\pi_0(k'))$, individually multiplied by the interference channel between the BS $A_{i'}(\pi_0^t(k'))$

assigned to each element $i' \in \Gamma(\pi_0(k'))$ and our typical user $i = 0$ at resource element k'. Here, we will assume that the optimal power strategy used by any user $i' \in \Gamma(\pi_0(k'))$ does not depend on the interference channel between his assigned BS $A_{i'}(\pi_0^t(k'))$, but is only conditioned by the transmission channel $h_{A_{i'}(\pi_0^t(k'),i'}(\pi_0^t(k'))$. Namely, the optimal power strategies and the interference channels are decorrelated: this allows to separate the expectation of the product as the product of the expectations. We can then respectively introduce h_0^{int} and p_0^{int}, the average interference channels and transmission powers used by elements in set $\Gamma(\pi_0(k'))$. Part (A) then rewrites as:

$$(A) = |\Gamma(\pi_0(k'))| \mathbb{E}\left[h_0^{int}\right] \mathbb{E}\left[p_0^{int}\right] \tag{37}$$

Where:

$$\begin{aligned} \mathbb{E}\left[h_0^{int}\right] &= \frac{1}{|\Gamma(\pi_0(k'))|} \sum_{i' \in \Gamma(\pi_0(k'))} \mathbb{E}\left[h_{A'_i(\pi_0^t(k')),0}(\pi_0^t(k'))\right] \\ \mathbb{E}\left[p_0^{int}\right] &= \frac{1}{|\Gamma(\pi_0(k'))|} \sum_{i' \in \Gamma(\pi_0(k'))} \mathbb{E}\left[\tilde{p}(\pi_{i'}^{-1}(\pi_0(k'))), Y_{i'}(\pi_{i'}^{-1}(\pi_0(k')))\right] \end{aligned} \tag{38}$$

$\mathbb{E}\left[p_0^{int}\right]$ can be expressed using notations introduced in the FPK section, as follows:

$$\mathbb{E}\left[p_0^{int}\right] = \sum_{i' \in \Gamma(\pi_0(k'))} \int_{Y_{i'}(\theta_{0,i'}^k) \in \mathcal{Y}} M_{i'}^{Y_{i'}(\theta_{0,i'}^k)}(\theta_{0,i'}^{k'}, Y_{i'}(\theta_{0,i'}^{k'}))\tilde{p}(\theta_{0,i'}^{k'}, Y_{i'}(\theta_{0,i'}^{k'}))dY_{i'}(\theta_{0,i'}^k) \tag{39}$$

Where $\theta_{0,i'}^k = \pi_{i'}^{-1}(\pi_0(k))$. The average interference channel can be approximated using stochastic geometry. In particular, the average interference channel between our typical user and a BS in the system can be expressed as follows:

$$\begin{aligned} \mathbb{E}\left[h_0^{int}\right] &= \lambda\pi\Delta_d^2 \int_{\theta=0}^{2\pi} \int_{r=0}^{\Delta_d} \frac{1}{\pi\Delta_d^2} \max\left(1, \frac{1}{r^\beta}\right) r dr d\theta \\ &= 2\pi^2\lambda \int_{r=0}^{\Delta_d} \max\left(1, \frac{1}{r^\beta}\right) r dr \\ &= 2\pi^2\lambda \left(\int_{r=0}^{1} r dr + \int_{r=1}^{\Delta_d} \frac{1}{r^{\beta-1}}\right) \\ &= \lambda\left[\pi + \frac{2\pi}{\beta-2}\left(1 - \frac{1}{\Delta_d^{\beta-2}}\right)\right] \end{aligned} \tag{40}$$

The convergence of the MF interference can be established by realizing the terms $\mathbb{E}\left[h_0^{int}\right]$ remains bounded when the number of elements goes large, i.e. $\Delta_d \to \infty$. The average power $\mathbb{E}\left[p_0^{int}\right]$ is simply bounded by P^{max}, and $|\Gamma(\pi_0(k'))|$ is finite, since we have assumed that the BS-UE assignments and resource allocation were known in advance. In particular, the values of $|\Gamma(\pi_0(k'))|$ are closely related to the average number of active users in the system, that we previously introduced as $\frac{\mu_{st}}{\mu_{at}}$. □

References

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