Elements of Proof: Interference Empowered 5G Networks

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***Abstract*—The following paper contains informative elements of proof related to the conference paper *"Interference Empowered 5G Networks"* [1], submitted to 5GU 2015.**

I. Appendix A: Proof of Proposition I.1

Proposition I.1. *For any given channel realization Γ and any configuration inducing orthogonalization, there exists a configuration, with no orthogonalization that outperforms it. More precisely:*

- $(2,2)$ *is outperformed by either* $(1,3)$, $(3,1)$.
- $(2,2)^*$ *is outperformed by either* $(1,3)^*$, $(3,1)^*$.
- $(2,2)^1$ *is outperformed by either* $(2,3)^*$, $(3,2)$.
- $(2,2)^2$ *is outperformed by either* $(3,2)^*$, $(2,3)$.

Note that $(2,3)$ *and* $(3,2)$ *were regimes defined in [2][3]. Also,* $(2,3)^*$ *and* $(3,2)^*$ *are just symmetric versions of it. Details about these configurations are given in proof.*

Elements of proof for the first statement can be found in [3] and [4]. In a symmetric way, the second statement can be demonstrated.

Let us now recall $(2,3)$ and $(3,2)$, two regimes defined in [2][3]. Also, $(2,3)^*$ and $(3,2)^*$ are just symmetric versions of it. On one side, the interferer is able to decode the interference and cancel it out via SIC techniques. This first interferer transmits at its point-to-point channel capacity. On the other side, the interferer only transmits using half of the spectral resources, and is affected by interference. In [2][3], we have shown that these two regimes were outperformed by either $(3,1)$ or $(1,3)$. Let us now define the performance of $(3,2)$ and $(2,3)^*$ as:

$$R((3,2),\Gamma) = \log_2\Big(1+\gamma(1,1)\Big)$$
$$+\frac{1}{2}\min\left[\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right),\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)\right]$$
$$R((2,3),\Gamma)^* \log_2\Big(1+\gamma(1,2)\Big)$$
$$+\frac{1}{2}\min\left[\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right),\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)\right]$$

It is easy to verify that a sufficient condition for $(3,2) \triangleright (2,2)^1$ is $\gamma(1,1) \geq \gamma(1,2)$. The same way, a sufficient condition for $(2,3)^* \triangleright (2,2)^1$ is $\gamma(1,1) \leq \gamma(1,2)$. From this, we easily show that $(2,2)^1$ is always outperformed by either $(3,2)$ or $(2,3)^*$. Note that we have also shown, in [3] and [4], that for any channel configuration Γ, the regimes $(2,3)$ and $(3,2)$ were outperformed by $(1,3)$ and $(3,1)$.

In a symmetric way, we can demonstrate that $(2,2)^2$ is outperformed by either $(3,2)^*$, $(2,3)$.

II. Appendix B: Proof of Proposition II.1

Proposition II.1. *When both users wish to treat interference using SIC-based techniques, i.e.* $\mathcal{O} = (3,3)$*, it is more interesting for the system to transmit using the interfering links, instead of its assigned ones. Id est,* $\forall\Gamma$*:*

$(3,3)$ *is outperformed by* $(1,1)^*$.
$(3,3)^*$ *is outperformed by* $(1,1)$.

Let us first define the spectral efficiencies associated to $(3,3)$ and $(1,1)^*$, as:

$$R((1,1)^*,\Gamma) = \log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)+\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$$

$$R((3,3),\Gamma) = \log_2\left(1+\min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$$

$$+\log_2\left(1+\min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$$

Since

$$\begin{cases} \min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right] \leq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\ \min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right] \leq \frac{\gamma(1,2)}{1+\gamma(2,2)} \end{cases}$$

, it comes immediately that $R((1,1)^*,\Gamma) \geq R((3,3),\Gamma)$, i.e. $(1,1)^* \triangleright (3,3)$ In a symmetric way, the second part of the statement can be demonstrated.

III. Appendix C: Proof of Proposition III.1

Proposition III.1. *We define the "6-Regimes Configuration Classifier", as follows.*

1) If $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$

- $(1,1)$ *BPC* $\Leftrightarrow$ $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$ *and* $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$
- $(1,3)$ *BPC* $\Leftrightarrow$ $(1,1)$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)$
- $(3,1)$ *BPC* $\Leftrightarrow$ $(1,1)$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1)$

2) If $\gamma(1,1) \leq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$

- $(1,1)^*$ *BPC* $\Leftrightarrow$ $\gamma(2,1) \geq \gamma(2,2)(1+\gamma(1,1))$ *and* $\gamma(1,2) \geq \gamma(1,1)(1+\gamma(2,2))$
- $(1,3)^*$ *BPC* $\Leftrightarrow$ $(1,1)^*$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)$
- $(3,1)^*$ *BPC* $\Leftrightarrow$ $(1,1)^*$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1)$

3) If $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$

- $(3,1)$ *BPC*

$$\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2) \geq (1+\gamma(2,1))(1+\gamma(1,2))$$

- $(3,1)^*$ *BPC*

$$\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2) \leq (1+\gamma(2,1))(1+\gamma(1,2))$$

4) If $\gamma(1,1) \leq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$

- $(1,3)^*$ *BPC*

$$\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2) \leq (1+\gamma(2,1))(1+\gamma(1,2))$$

- $(1,3)$ *BPC*

$$\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2) \geq (1+\gamma(2,1))(1+\gamma(1,2))$$

We decompose the proof of this proposition, in 4 subsections, one for each part of the proposition.

A. Proof of 1) in Proposition III.1

When $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$, we have, according to Propositions IV.1 and V.1:

$$\begin{cases} (1,1) \triangleright (1,1)^* \\ (3,1) \triangleright (1,3)^* \\ (1,3) \triangleright (3,1)^* \end{cases}$$

And, only 3 configurations can pretend to be the Best Performing Configuration (BPC), when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$: $(1,1)$, $(1,3)$ and $(3,1)$.

At first, it is easy to verify, using Proposition VI.1, that $(1,1)$ is the best performing configuration (BPC) if and only if $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$. The first statement is set.

When $(1,1)$ is not the BPC, i.e. $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ or $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$, we may identify 3 cases and confront the two remaining configurations : $(1,3)$ and $(3,1)$.

TABLE I

12 admissible configurations $\mathcal{O}$ and their spectral efficiencies performances

$\mathcal{O}$	$R_1(\mathcal{O},\Gamma)$	$R_2(\mathcal{O},\Gamma)$	$R(\mathcal{O},\Gamma) = R_1(\mathcal{O},\Gamma) + R_2(\mathcal{O},\Gamma)$
$(1,1)$	$\log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)$	$\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)$	$\log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)+\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)$
$(2,2)$	$\frac{1}{2}\log_2\left(1+\gamma(1,1)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(2,2)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(1,1)\right)+\frac{1}{2}\log_2\left(1+\gamma(2,2)\right)$
$(3,1)$	$\log_2\left(1+\gamma(1,1)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$	$\log_2\left(1+\gamma(1,1)\right)+\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$
$(1,3)$	$\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$	$\log_2\left(1+\gamma(2,2)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)+\log_2\left(1+\gamma(2,2)\right)$
$(3,3)$	$\log_2\left(1+\min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$ $+\log_2\left(1+\min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$
$(1,1)^*$	$\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)$	$\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$	$\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)+\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$
$(2,2)^*$	$\frac{1}{2}\log_2\left(1+\gamma(2,1)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(1,2)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(2,1)\right)+\frac{1}{2}\log_2\left(1+\gamma(1,2)\right)$
$(3,1)^*$	$\log_2\left(1+\gamma(2,1)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$	$\log_2\left(1+\gamma(2,1)\right)+\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$
$(1,3)^*$	$\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$	$\log_2\left(1+\gamma(1,2)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)+\log_2\left(1+\gamma(1,2)\right)$
$(3,3)^*$	$\log_2\left(1+\min\left[\gamma(2,1),\frac{\gamma(2,2)}{1+\gamma(1,2)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(1,2),\frac{\gamma(1,1)}{1+\gamma(2,1)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(2,1),\frac{\gamma(2,2)}{1+\gamma(1,2)}\right]\right)$ $+\log_2\left(1+\min\left[\gamma(1,2),\frac{\gamma(1,1)}{1+\gamma(2,1)}\right]\right)$
$(2,2)^1$	$\log_2\left(1+\gamma(1,1)\right)$	$\log_2\left(1+\gamma(1,2)\right)$	$\log_2\left(1+\gamma(1,1)\right)+\log_2\left(1+\gamma(1,2)\right)$
$(2,2)^2$	$\log_2\left(1+\gamma(2,1)\right)$	$\log_2\left(1+\gamma(2,2)\right)$	$\log_2\left(1+\gamma(2,1)\right)+\log_2\left(1+\gamma(2,2)\right)$

- When $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$, we have, according to Proposition VI.1, $(1,1) \triangleright (3,1)$ and $(1,3) \triangleright (1,1)$. This immediately leads to $(1,3)$ BPC.
- When $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$, we have, according to Proposition VI.1, $(1,1) \triangleright (1,3)$ and $(3,1) \triangleright (1,1)$. This immediately leads to $(3,1)$ BPC.

One case remains: $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$ and $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$. It is easy to verify that under these conditions, we have $\frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}$ and $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$.

It immediately follows:

$$\begin{aligned}
&(1,3) \triangleright (3,1)\\
&\Leftrightarrow \log_2(1+\gamma(2,2)+\gamma(1,2)) \geq \log_2(1+\gamma(1,1)+\gamma(2,1))\\
&\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)
\end{aligned}$$

Summing up, $(1,3)$ is BPC when

$$\left\{\begin{array}{l}
\gamma(1,1) \geq \gamma(1,2)\\
\text{and } \gamma(2,2) \geq \gamma(2,1)\\
\text{and } \gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))\\
\text{and } [\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))\\
\text{or } \Big(\ \gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))\\
\ \text{and }\ \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)\Big)\Big]
\end{array}\right.$$

Also, when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$, we have:

$$\begin{cases}
\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2)) \text{ and } \gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))\\
\Rightarrow \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)
\end{cases}$$
$$\begin{cases}
\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2)) \text{ and } \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))\\
\Rightarrow \gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1)
\end{cases}$$

This leads to $(1,3)$ is BPC when

$$\left\{\begin{array}{l}
\gamma(1,1) \geq \gamma(1,2)\\
\text{and } \gamma(2,2) \geq \gamma(2,1)\\
\text{and } (1,1) \text{ not BPC}\\
\text{and } \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)
\end{array}\right.$$

This corresponds exactly to the second statement. The same way, we define the BPC conditions for $(3,1)$ and conclude the proof.

B. Proof of 2) in Proposition III.1

We may proceed as in Subsection III-A and exploit symmetric properties.

C. Proof of 3) in Proposition III.1

When $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we have, according to Propositions IV.1 and V.1:

$$\left\{\begin{array}{l}
\gamma(2,1)(1+\gamma(1,2)) \geq \gamma(2,2)\\
\gamma(1,1)(1+\gamma(2,2)) \geq \gamma(1,2)\\
(3,1)^* \triangleright (1,1)^*\\
(3,1) \triangleright (1,1)\\
(3,1) \triangleright (1,3)^*\\
(3,1)^* \triangleright (1,3)
\end{array}\right.$$

At this point, only 2 configurations can pretend to be the Best Performing Configuration (BPC), when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$: $(3,1)^*$ and $(3,1)$. Also, note that when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we have necessarily $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ or $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$.

We may now identify 3 cases and confront the two remaining configurations : $(1,3)$ and $(3,1)$.

1) If $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, we have:

$$\begin{aligned}
&(3,1) \triangleright (3,1)^*\\
&\Leftrightarrow R((3,1),\Gamma) \geq R((3,1)^*,\Gamma)\\
&\Leftrightarrow log_2(1+\gamma(1,1)) + \log_2\left(1+\tfrac{\gamma(2,1)}{1+\gamma(1,1)}\right)\\
&-\log_2(1+\gamma(2,1)) - \log_2\left(1+\tfrac{\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0\\
&\leftrightarrow \log_2\left(1+\tfrac{\gamma(1,1)}{1+\gamma(2,1)}\right) - - \log_2\left(1+\tfrac{\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0\\
&\Leftrightarrow \tfrac{\gamma(1,1)}{1+\gamma(2,1)} \geq \tfrac{\gamma(1,2)}{1+\gamma(2,2)}
\end{aligned}$$

Which is true, by definition. In this case, $(3,1)$ is BPC.

2) The same way, if $\frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, we have:

$$\begin{aligned}
&(3,1) \triangleright (3,1)^*\\
&\Leftrightarrow \tfrac{\gamma(2,2)}{1+\gamma(1,2)} \geq \tfrac{\gamma(2,1)}{1+\gamma(1,1)}
\end{aligned}$$

Which is not true, by definition. In this case, $(3,1)^*$ is BPC.

3) If $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, it follows:

$$\begin{aligned}
&(3,1) \triangleright (3,1)^*\\
&\Leftrightarrow \log_2\left(1+\tfrac{1+\gamma(1,1)}{1+\gamma(2,1)}\right) - \log_2\left(1+\tfrac{1+\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0\\
&\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))
\end{aligned}$$

To sum up, $(3,1)$ is BPC if

$$\begin{array}{l}
\gamma(1,1) \geq \gamma(1,2)\\
\text{and } \gamma(2,2) \geq \gamma(2,1)\\
\text{and } \left[\left[\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right.\\
\text{or } \left[\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}\ \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}\right.\\
\text{and }\ (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))]]
\end{array}$$

That can be simplified into: $(3,1)$ is BPC if

$$\begin{array}{l}
\gamma(1,1) \geq \gamma(1,2)\\
\text{and } \gamma(2,2) \geq \gamma(2,1)\\
\text{and } \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}\\
\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))
\end{array}$$

The same way: $(3,1)^*$ is BPC if

$$\begin{array}{l}
\gamma(1,1) \geq \gamma(1,2)\\
\text{and } \gamma(2,2) \geq \gamma(2,1)\\
\text{and } \frac{\gamma(2,1)}{1+\gamma(1,1)} \geq \frac{\gamma(2,2)}{1+\gamma(1,2)}\\
\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2))
\end{array}$$

Finally, when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we necessarily have:

$$\left\{ \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ or } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)} \right.$$

$$\left\{ \begin{array}{l} \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\ \Rightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2)) \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\ \Rightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2)) \end{array} \right.$$

And this leads immediately to That can be simplified into: $(3,1)$ is BPC if

$$\begin{array}{l} \gamma(1,1) \geq \gamma(1,2) \\ \text{and } \gamma(2,2) \geq \gamma(2,1) \\ \text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2)) \end{array}$$

And $(3,1)^*$ is BPC if

$$\begin{array}{l} \gamma(1,1) \geq \gamma(1,2) \\ \text{and } \gamma(2,2) \geq \gamma(2,1) \\ \text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2)) \end{array}$$

D. Proof of 4) in Proposition III.1

We may proceed as in Subsection III-C and exploit symmetric properties.

IV. Appendix D: Proof of Proposition IV.1

Proposition IV.1. *The following statements hold*

- $(1,3) \triangleright (3,1)^* \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)$
- $(3,1) \triangleright (1,3)^* \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)$

Based on Table I, it immediately comes, $(1,3) \triangleright (3,1)^* \Leftrightarrow R((1,3),\Gamma) \geq R((3,1)^*,\Gamma) \Leftrightarrow \log_2\Big(1+\gamma(2,2)\Big) \geq \log_2\Big(1+\gamma(2,1)\Big) \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)$.
The same way, we can show that $(3,1) \triangleright (1,3)^* \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)$.

V. Appendix E: Proof of Proposition V.1

Proposition V.1. *The following statements hold*

- $(1,1) \triangleright (1,1)^* \Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(1,2))(1+\gamma(2,1))$.
- *A sufficient condition for* $(1,1) \triangleright (1,1)^*$ *is* $\gamma(1,1) \geq \gamma(1,2)$ *and* $\gamma(2,2) \geq \gamma(2,1)$.
- *A sufficient condition for* $(1,1)^* \triangleright (1,1)$ *is* $\gamma(1,1) \leq \gamma(1,2)$ *and* $\gamma(2,2) \leq \gamma(2,1)$.

Based on Table I, it immediately comes,

$$\begin{array}{l} (1,3) \triangleright (3,1)^* \\ \Leftrightarrow R((1,1),\Gamma) \geq R((1,1)^*,\Gamma) \\ \Leftrightarrow \log_2\left(\frac{(1+\gamma(1,1)+\gamma(2,1))(1+\gamma(2,2)+\gamma(1,2))}{(1+\gamma(2,1))(1+\gamma(1,2))}\right) \\ -\log_2\left(\frac{(1+\gamma(1,1)+\gamma(2,1))(1+\gamma(2,2)+\gamma(1,2))}{(1+\gamma(1,1))(1+\gamma(2,2))}\right) \geq 0 \\ \Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(1,2))(1+\gamma(2,1)) \end{array}$$

Then, it is easy to verify that $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$ (resp. $\gamma(1,1) \leq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$) are sufficient conditions for $(1,1) \triangleright (1,1)^*$ (resp. $(1,1)^* \triangleright (1,1)$).

VI. Appendix F: Proof of Proposition VI.1

Proposition VI.1. *The following statements hold*

- $(1,1) \triangleright (1,3) \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$.
- *A sufficient condition for* $(1,1) \triangleright (1,3)$ *is* $\gamma(1,1) \geq \gamma(1,2)$.
- $(1,1) \triangleright (3,1) \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$.
- *A sufficient condition for* $(1,1) \triangleright (3,1)$ *is* $\gamma(1,1) \geq \gamma(2,1)$.
- $(1,1)^* \triangleright (1,3)^* \Leftrightarrow \gamma(2,1) \geq \gamma(2,2)(1+\gamma(1,1))$.
- *A sufficient condition for* $(1,1)^* \triangleright (1,3)^*$ *is* $\gamma(2,1) \geq \gamma(2,2)$.
- $(1,1)^* \triangleright (3,1)^* \Leftrightarrow \gamma(1,2) \geq \gamma(1,1)(1+\gamma(2,2))$.
- *A sufficient condition for* $(1,1)^* \triangleright (3,1)^*$ *is* $\gamma(1,2) \geq \gamma(1,1)$.

Based on Table I, we have:

$$\begin{array}{l} (1,1) \triangleright (1,3) \\ \Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right) + \log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right) \\ -\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)}, \frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right) + \log_2(1+\gamma(2,2)) \geq 0 \end{array}$$

Note that, if $\frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)}$, then we can not have $(1,1) \triangleright (1,3)$. Necessarily $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)}, \frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right) = \log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$. It follows:

$$\begin{array}{l} (1,1) \triangleright (1,3) \\ \Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right) + \log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right) \\ -\log_2(1+\gamma(2,2)+\gamma(1,2)) \geq 0 \\ \Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right) - \log_2(1+\gamma(1,2)) \geq 0 \\ \Leftrightarrow \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \gamma(1,2) \end{array}$$

Also, it is easy to verify that $\gamma(1,1) \geq \gamma(1,2) \Rightarrow \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1)) \Leftrightarrow (1,1) \triangleright (1,3)$.

The same way, using symmetric properties, the 6 other statements can be demonstrated.

VII. Appendix G: Proof of Proposition VII.1

Proposition VII.1. *A sufficient condition for* $(1,3) \triangleright (3,1)$ *is* $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$ *and* $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$.

Refer to [4]. Also, by transitivity from Proposition VI.1.

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