Energy-Efficient Proactive Scheduling in Ultra Dense Networks

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***Abstract*—In this paper, we investigate the energy efficiency (EE) performance of optimal scheduling strategies in ultra-dense networks (UDNs). We consider a network deployment of base stations (BSs) based on a homogeneous Poisson Point Process (PPP), and assume users requests, modeled according to a space-time homogeneous Point Process (STPP), are to be served within a given service time. The objective is to define the optimal scheduling strategy, that allows to serve every request during its required service time, while minimizing the energy consumed in the process. The optimization consists of a Dynamic Stochastic Game (DSG), which is hard to solve in the UDNs context, due to the coupling of interference, the large number of elements interacting, as well as uncertainties on the channel dynamics, interference and future requests. Our contribution lies in addressing the inherent complexity issue of the DSG, by transitioning into an equivalent and more tractable Mean Field Game (MFG). By combining the MFG framework with elements of stochastic geometry and queuing theory, the analysis of the optimal scheduling strategies is then conducted. The provided numerical simulations give good insights on notable performance gains, in terms of EE.**

I. Introduction

Mobile networks densification and Ultra-Dense Networks (UDNs) are currently viewed as two key enabling solutions, for meeting an ever-increasing capacity demand, while minimizing at the same time the network expenditures [1]. In such a UDN context with significant amounts of transmitters and receivers, optimization approaches, such as power control, scheduling and other management techniques become significantly challenging: the bottleneck lies in the coupling of interference over a large number of elements, as well as channel uncertainties and fluctuations on future requests. However difficult, power control and scheduling have been recently investigated in numerous works, often cast as Dynamic Stochastic Games (DSGs) [2], [3], which are inadequate in a UDN context due to their inherent mathematical complexity. As a consequence, most works in literature at the moment are either left practically unsolved, are based on heuristics, or consist of simulations that lack fundamental insights. However, Mean Field Games (MFGs) have recently received attention, as they allow to address the inherent mathematical complexity of DSGs, by turning the initial DSG into a more tractable Mean Field Game [4], [5]: this equivalent MFG allows for drastic simplifications, by addressing the coupling issue on interference. The trick was made possible by considering symmetries between users, which allows to simplify the global interactions between users, by re-expressing interference as a Mean Field interference perceived by all users within the network. Power control and scheduling problems have then been investigated, in multiple papers. In [6], the authors used MFGs to adapt the transmission powers, to the battery level and channel links, in order to meet a required SINR threshold. In [7], the authors investigate the scheduling problem, which consists of serving multiple interfering users, from different cells, at the same time and at a minimal power cost. In [8], a joint power control and user scheduling combining MFGs with Lyapunov optimization is proposed, and allows to enhance the energy efficiency of the network. The most recent trend for MFGs in wireless networks, consists of understanding how coupling MFGs with Stochastic Geometry could help express a more accurate Mean Field interference [9].

In this paper and as detailed in Section II, we consider a network deployment of base stations (BSs) based on a homogeneous Poisson Point Process (PPP). We also assume users requests, whose arrivals, service times and data requests are modeled according to a space-time homogeneous Point Process (STPP), are to be served within a given service time. Here, the objective is to define the optimal scheduling strategy, that allows to serve every request during its required service time, while minimizing the energy consumed in the process. The underlying optimization consists of a DSG, which is hard to solve in a ultra dense network context, due to the large number of elements in the network, the numerous interactions that one has to take into account in the coupling of interference, as well as uncertainties on the channel dynamics and future requests. We detail the DSG at stake in Section IV and show where the impassable mathematical complexity lies. Our contribution lies in transitioning the DSG into a more tractable Mean Field Game, whose detailed analysis is given in Section V. The analysis of the optimal scheduling strategies is then conducted, by combining the MFG framework with elements of stochastic geometry and queuing theory: combining MFGs and Stochastic geometry allows for a more accurate definition of the Mean Field interference term, as it allows to take into account both spatial and temporal dynamics [9]. Finally, we conclude the paper, by providing numerical simulations in Section VI, demonstrating a notable performance gain, in terms of Energy Efficiency (EE), between the optimal scheduling strategy obtained through the proposed MFG approach, and a state-of-the-art heuristic, namely the equal-bit scheduler [10].

II. System Model

In this section, we introduce the different aspects of the system model considered throughout this paper.

1) Base Stations Deployment: We consider a geographic zone consisting of a circle of radius Δ_d. As in [11], the deployment of BSs is modeled according to a homogeneous Poisson Point Process (PPP), with a spatial density λ_b BSs per km^2. In the following, we will denote $\bar{\mathcal{M}}$, the set of BSs in the system, that will be indexed with $j \in \bar{\mathcal{M}} = [\![1, |\bar{\mathcal{M}}|]\!]$.

2) User requests: We consider a discrete representation of time, and run the simulation over T time slots of duration Δ_t, indexed by $t \in \bar{\mathcal{T}} = [\![1, T]\!]$. The users requests are modeled according to a Space-Time Point Process (STPP). At the beginning of the first time slot, a first user request arrives. The inter-arrival times for the following requests are modeled as an exponential distribution with parameter μ_{at}. Each request is to be served by the system as soon as it arrives, and the service time, follows an exponential distribution with parameter μ_{st}. For every request/user, indexed by $i \in \bar{\mathcal{N}} = [\![1, |\bar{\mathcal{N}}|]\!]$, we will denote $\bar{\mathcal{T}}_i = \left\{t \in \bar{\mathcal{T}} \mid T_i^s \leq t \leq T_i^e\right\}$, the service time for user i, with T_i^s the time slot corresponding to the arrival of the request and T_i^e the last time slot for service. Each user arrives with a data request whose value D_i^r is modeled according to a random exponential process with parameter μ_{dr}.

3) Users Mobility and Propagation model: Let us denote $h_{j,i}(t)$ the channel gain between BS j and user i, which remains constant during time slot t. In this paper, we assume, that the channels can be modeled according to a deterministic path loss model, based on the Euclidean distance $d_{j,i}(t)$ between the BS j and user i at time t. The considered path loss function is in the form of:

$$h_{j,i}(t) = \min\left(1, \left(\frac{1}{d_{j,i}(t)}\right)^{\beta}\right) \tag{1}$$

Where β is the path loss exponent for path loss. We also assume users are mobile, and that the channel evolution, resulting from the users mobility, can then be represented as a Stochastic Differential Equation (SDE) [3], [12], following:

$$h_{j,i}(t+1) = h_{j,i}(t) + \alpha_{j,i}(t, h_{j,i}(t))\Delta_t + \sigma^b_{j,i}(t)\mathcal{B}_{j,i}(t) \tag{2}$$

With $\mathcal{B}_{j,i}(t)$ a Brownian motion, modeled as $\mathcal{N}(0, \Delta_t)$. The deterministic part account for the path loss evolution (due to mobility) and channel evolution predictions, whereas the stochastic part can be used to model fading and prediction errors/uncertainty on the users mobility [6], [7].

In this paper, we assume that there is a UE-BS assignment procedure, to allocate users to BSs at every time slot of their active service time. Furthermore, we assume a resource allocation is performed, allocating users from a same BS to the bandwidth elements available for transmission, with no two users from a same BS assigned to the same bandwidth element at the same time. For the sake of simplicity, both procedures are based on [11] and rely on the following two hypotheses.

4) BS-UE assignment strategy: The users can only be assigned to one BS at each time slot. At each time slot t, each user i is assigned to the BS that is the geographically closest BS, denoted $A_i(t)$, i.e. $\forall t \in \bar{\mathcal{T}}_i, A_i(t) = \mathrm{argmin}_{j \in \bar{\mathcal{M}}}\left[d_{j,i}(t)\right]$.

5) Bandwidth (BW) allocation: Once the BS-UE assignments are defined, the available BW at each BS, noted $\bar{\mathcal{W}}$ is equally split between all users assigned to the BS at a given time t. This is easily obtained via equal TDD/FDD for example [13]. We assume the BW is split in W equal parts of size Δ_w, indexed by $w \in \bar{\mathcal{W}} = [\![1, W]\!]$. We can then define the BW assigned to user i at time t, by $\bar{\mathcal{W}}_i(t)$, where $\bar{\mathcal{W}}_i(t)$ is a subset of $\bar{\mathcal{W}}$ containing the indexes of the bandwidth elements assigned to user i at time t.

Note that the presented work applies for any BS-UE assignments and BW allocation procedures. In this paper, we assume the system has perfect a priori knowledge of the requests (arrival times, service times and data requests). The system also has perfect a priori knowledge of the future AP-UE assignments, and future bandwidth allocation. This strong hypothesis will be relaxed in future works.

III. Energy-Efficient Scheduling Optimization Problem

In this paper, the objective consists of finding the optimal scheduling strategy for every user i, $\mathcal{P}_i^* = (p_i^*(t,w))_{t \in \bar{\mathcal{T}}_i, w \in \bar{\mathcal{W}}_i(t)}$, that allows to transmit the required data packet D_i^r before the service time ends, at a minimal energy consumption cost $E_i^*(\mathcal{P}_i^*) = \sum\limits_{\substack{t \in \bar{\mathcal{T}}_i \\ w \in \bar{\mathcal{W}}_i(t)}} p_i^*(t,w)\Delta_t$. With this notation, the BS $A_i(t)$ assigned to user i, transmits at power $p_i(t,w) > 0$, at time slot t and on BW element w. Non-assigned BSs, become dormant. Before we introduce the optimization problem at stake, let us first introduce $K_i = [\![1, |K_i| = \sum\limits_{t \in \bar{\mathcal{T}}_i} |\bar{\mathcal{W}}_i(t)|]\!]$ and the bijective function π_i, which is defined as $\pi_i : k \rightarrow (\pi_i^t(k), \pi_i^w(k)) = (t,w)$. In fact, the π_i function transforms (t,w) into a single resource element (RE) index $k \in K_i$, which carries the information about the time slot index ($t = \pi_i^t(k)$), the bandwidth element index ($w = \pi_i^w(k)$), as well as the amount of remaining REs available for transmission before the end of service time for user i (which is simply k). This transformation allows to simplify the equations to be considered for optimization. Also, the π_i function is bijective and verifies:

$$\forall k, k' \in K_i, k > k' \Leftrightarrow \begin{cases} \pi_i^t(k) < \pi_i^t(k') \\ \text{or } \pi_i^t(k) = \pi_i^t(k') \text{ and } \pi_i^w(k) < \pi_i^w(k') \end{cases} \tag{3}$$

The optimal power strategy, now writes $\mathcal{P}_i^* = (p_i^*(k))_{k \in K_i}$ and satisfies to the following minimal power consumption optimization (4), under service constraints (5)-(10):

$$\mathcal{P}_i^* = \underset{\mathcal{P}_i}{\mathrm{argmin}}\left[\sum_{k \in K_i} p_i(k)\Delta_t\right] \tag{4}$$

The transmission constraint consists of ensuring a complete transmission of the requested data packet of any user i, before the end of its service time. Similarly to [7], we denote $Q_i(k)$, the remaining data packet at the beginning of the k-th remaining RE, and thus, we have, $k \in K_i$:

$$Q_i(k-1) = Q_i(k) - \log_2\left(1 + \frac{p_i(k) h_{A_i(\pi_i^t(k)),j}(\pi_i^t(k))}{\sigma_n^2 + I_i(\pi_i(k))}\right)\Delta_w \Delta_t \tag{5}$$

The initial value of the queue state verifies $Q_i(|K_i|) = D_i^r$, and a complete transmission, then verifies:

$$Q_i(0) = D_i^r - \sum_{k \in K_i} \log_2\left(1 + \frac{p_i(k) h_{A_i(\pi_i^t(k)),j}(\pi_i^t(k))}{\sigma_n^2 + I_i(\pi_i(k))}\right)\Delta_w \Delta_t = 0 \tag{6}$$

The interference term $I_i(t,w)$ consists of the transmissions powers of other antennas, happening at the same time t on the same bandwidth element w, and then writes:

$$I_i(t,w) = \sum_{\substack{i' \in \mathcal{N} \\ i' \neq i}} \delta(w \in \bar{\mathcal{W}}_{i'}(t)) p_{i'}(t,w) h_{A_{i'}(t),i}(t) \tag{7}$$

With $\delta(A)$, the Kroenecker symbol, equal to 1 if statement A is true, and 0 otherwise. Last but not least, the channel gain evolution model was defined, for all possible channels, by (2). We now only consider the channel gain $h_i^a(k)$, used for transmission by user i on RE k. To do so, let us first introduce the subset K_i^l, which contains the indexes in K_i which verify that $\pi_i^w(k)$ is the last bandwidth element assigned to user i on time slot $\pi_i^t(k)$. It is defined as $K_i^l = \left\{ k \in K_i \mid k = \text{argmax}_{\substack{k' \in K_i \\ \pi_i^t(k') = \pi_i^t(k)}} [\pi_i^w(k')] \right\}$. We can then rewrite the evolution of the channel assigned to user i, as follows:

$$h_i^a(k-1) = h_i^a(k) + \alpha^a(k)\Delta_t + \sigma_i^{a,b}(k)\mathcal{B}_i^a(k) \tag{8}$$

Where $\alpha^a(k) = 0$ if $k \notin K_i^l$, and otherwise consists of the deterministic prediction about the future channel evolution, which takes into account the possible handovers, as well, i.e.

$$\begin{aligned}
\alpha^a(k) = &\left[h_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(|K_i|)) - h_{A_i(\pi_i^t(k)),i}(\pi_i^t(|K_i|))\right]\tfrac{1}{\Delta_t} \\
&+ \sum_{k'=k+1}^{|K_i|} \Bigg[\alpha_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k')) + \frac{\sigma^b_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k'))}{\Delta_t}\mathcal{B}_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k')) \\
&- \alpha_{A_i(\pi_i^t(k)),i}(\pi_i^t(k')) - \frac{\sigma^b_{A_i(\pi_i^t(k)),i}(\pi_i^t(k'))}{\Delta_t}\mathcal{B}_{A_i(\pi_i^t(k)),i}(\pi_i^t(k'))\Bigg] \\
&+\alpha_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k))
\end{aligned} \tag{9}$$

Note that when no handover occurs (i.e. when we have $A_i(\pi_i^t(k-1)) = A_i(\pi_i^t(k))$), (9) simply consists of a single term $\alpha^a(k) = \alpha_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k))$. Similarly, we also have:

$$\sigma_i^{a,b}(k) = \begin{cases} \sigma_{A_i(\pi_i^t(k-1)),i}(\pi_i^t(k)) & \text{if } k \in K_i^l, \\ 0 & \text{else.} \end{cases} \tag{10}$$

And $\mathcal{B}_i^a(k)$ is a Brownian motion following $\mathcal{N}(0, \Delta_t)$. A complete proof can be found in [12].

IV. Multi-user Dynamic Stochastic Game Analysis and Limits of the Analysis

In this section, we focus on analyzing the multi-user Dynamic Stochastic Game (DSG) [2], [3], which consists of optimizing the energy consumption (4), under the constraints (5)-(10). At the beginning of each RE $k \in K_i$, the current channel realization for all the channels are revealed and the system must decide the transmission powers $p_i(k)$ to be used by user i on RE k. To do so, the system must balance the amount of data that is going to be transmitted on the current time slot, and what it leaves for the future. As a consequence, the optimal power strategy $p_i^*(k)$ to be used on the RE k, when user i is in state $X_i(k)$ then satisfies:

$$p_i(k)^* = \underset{p_i(k)}{\text{argmin}} \left[p_i(k)\Delta_t + V_i(k-1, X_i(k-1))\right] \tag{11}$$

Here, we have assumed for now that the system has an a priori estimate of the future interference $I_i(\pi_i(k))$, for user i and all $k \in K_i$. Also, we have $X_i(k) = [h_i^a(k), Q_i(k)]$. We must also introduce the Bellman function [2], $V_i(k-1, X_i(k-1))$, which consists of the expected energy consumption on the remaining $k-1$ REs, if user i arrives at RE $k-1$ in state $X_i(k-1)$, i.e.:

$$V_i(k-1, X_i(k-1)) = \sum_{\substack{k' \in K_i \\ k' \leq k-1}} \left(\mathbb{E}_{X_i(k')}\left[p_i^*(k)\Delta_t\right]\right) + \mathbb{E}\left[\epsilon(Q_i(0))\right] \tag{12}$$

Here, ϵ is a penalty function, relaxing constraint (6), as in [7], [8] which returns a significant penalty cost if $Q_i(0) > 0$ and no penalty if $Q_i(0) = 0$. We can define the optimal trajectory for the Bellman function V_i^* and the resulting optimal power strategies p_i^* to be used on each RE $k \in K_i$. The optimal Bellman function trajectory is solution to the HJB equation, detailed hereafter $\forall k' \in K_i, k' \leq k$:

$$\begin{aligned}
&\left[\frac{\partial V_i^*(k', X_i(k'))}{\partial Q_i(k')}\frac{\Delta_w}{\log(2)} - \frac{\sigma_n^2 + I_i(\pi_i(k'))}{h_i^a(k')}\right]^+ + \frac{\partial V_i^*(k', X_i(k'))}{\partial k'} \\
&- \log_{2+}\left(\frac{\partial V_i^*(k', X_i(k'))}{\partial Q_i(k')}\frac{\Delta_w}{\log(2)}\frac{h_i^a(k')}{\sigma_n^2 + I_i(\pi_i(k'))}\right)\Delta_w \frac{\partial V_i^*(k', X_i(k'))}{\partial Q_i(k')} \\
&+\alpha_i^a(k')\frac{\partial V_i^*(k', X_i(k'))}{\partial h_i^a(k')} + \frac{(\sigma_i^{b,a}(k'))^2}{2}\frac{\partial^2 V_i^*(k, X_i(k'))}{\partial h_i^a(k')^2} = 0
\end{aligned} \tag{13}$$

With terminal condition $V_i^*(0, X(0)) = \epsilon_i(Q_i(0))$. Here, we also use $[x]^+ = \min(P^{max}, \max(0,x))$ and $\log_{2+}(x) = \max(0, \log_2(x))$. The optimal power strategy p_i^* can then be derived from the optimal Bellman function V_i^*, as:

$$\forall k', p_i^*(k') = \left[\frac{\partial V_i^*(k', X_i(k'))}{\partial Q_i(k')}\frac{\Delta_w}{\log(2)} - \frac{\sigma_n^2 + I_i(\pi_i(k'))}{h_i^a(k')}\right]^+ \tag{14}$$

Details about this calculation can be found in [12]. At this point, the optimal solution for the system can be computed by an iterative algorithm, mimicking the iterative time-waterfilling algorithm [14]: i) pick a random user $i \in \mathcal{N}$; ii) solve its HJB equation to obtain the optimal power strategy p_i^*; iii) update the interference terms perceived by users $i' \in \mathcal{N}, i' \neq i$. The process goes on iteratively, until a convergence is observed on all users power strategies, thus indicating the Nash Equilibrium/optimal configuration has been reached. Even though this iterative process works, it is known to have a huge computational complexity, skyrocketing with the number of elements in the system [7], thus making the analysis of large-dimensional systems, such as Ultra-Dense Networks (UDNs), impossible in practice. To address this issue, we will transition into a Mean Field Game [4], [5], which helps simplify the analysis in the large case scenario. We detail this approach in the next section.

V. A MFG Approach to Scheduling, with Stochastic Geometry

In order to transition the optimization problem (4), into a Mean Field Game, we must consider 3 hypotheses that we detail hereafter.

Hyp. 1 - Large Number of Elements: The first hypothesis assumes, that the number of elements involved in the DSG (4), is large enough to be considered infinite. To do so, we formulate two hypotheses: i) Large geographical region, i.e. Δ_d grows large; and ii) The average inter-arrival time μ_{at} is modeled as a function of the region area, i.e. $\mu_{at} = \frac{\bar{\mu}_{at}}{\Delta_d^2}$. As a consequence, the average number of users active in the system at the same time, given by $\frac{\mu_{st}}{\mu_{at}} = \frac{\pi\Delta_d^2\mu_{st}}{\bar{\mu}_{at}}$ grows large with Δ_d. We can then model the spatial density of active users, as $\lambda_u = \frac{\mu_{st}}{\bar{\mu}_{at}}$.

Hyp. 2 - Indistinguishability Between Users: The second hypothesis consists of the indistinguishability property: the optimization problem (4) in Section IV, that a typical user i in the system is trying to solve is the same for every user, and only depends on a set of parameters $Y_i(k)$, available at the beginning of RE k:

- the set of its assigned resource elements K_i, and in particular depends on the amount of remaining resource elements (which is k);
- the current queue state of this user $Q_i(k)$;
- the current transmission channel state $h_i^a(k)$;
- the estimated future interference $I_i(\pi_i(k'))_{\substack{\forall k'\in K_i \\ k'\leq k}}$, which is assumed to be known to the user, a priori, and will later on, be estimated using elements of stochastic geometry and a common power strategy;
- the transmission channel gain evolution, which depends on parameters $(\alpha_i^a(k'))_{\substack{k'\in K_i \\ k'\leq k}}$ and $(\sigma_i^{b,a}(k'))_{\substack{k'\in K_i \\ k'\leq k}}$, which are again known in advance.

As a consequence, it makes sense to consider a unique optimization problem, whose outcome depends for each user i on the set of parameters $Y_i(k)$, available at RE k. If user $i = 0$ now denotes any typical user, whose parameters $Y_0(k)$ could be anything, we can define a unique Mean Field (MF) power strategy $\tilde{p}(k, Y_0(k))$, which applies to any user in the system, whose current state at $(t, w) = \pi_0(k)$ is $Y_0(k)$. The detailed proof of this MF transition, can be found in [12]. As in Section IV, the optimal MF power strategy is defined as the optimal solution to the optimization problem:

$$\tilde{p}(k, Y_0(k))^* = \underset{p}{\operatorname{argmin}} \left[p\Delta_t + \tilde{V}_0(k-1, Y_0(k-1)) \right] \quad (15)$$

Where the Bellman function $\tilde{V}_0$, now consists of:

$$\tilde{V}_0(k-1, Y_0(k-1)) = \sum_{\substack{k'\in K_i \\ k'\leq k-1}} \left(\mathbb{E}_{Y_0(k')}\left[p^*(k', Y_0(k'))\Delta_t\right] \right) + \mathbb{E}\left[\epsilon(Q_0(0))\right] \quad (16)$$

As before, the optimal Bellman function trajectory is solution to an HJB equation, similar to (13), $\forall k' \in K_0, k \leq k$:

$$\begin{aligned} &\left[\frac{\partial \tilde{V}_0^*(k', Y_0(k'))}{\partial Q_0(k')} \frac{\Delta_w}{\log(2)} - \frac{\sigma_n^2 + I_0(\pi_0(k'))}{h_0^a(k')} \right]^+ + \frac{\partial \tilde{V}_0^*(k', Y_0(k'))}{\partial k'} \\ &- \log_{2+}\left(\frac{\partial \tilde{V}_0^*(k', Y_0(k'))}{\partial Q_0(k')} \frac{\Delta_w}{\log(2)} \frac{h_0^a(k')}{\sigma_n^2 + I_0(\pi_0(k'))} \right) \Delta_w \frac{\partial \tilde{V}_0^*(k', Y_0(k'))}{\partial Q_0(k')} \\ &+ \alpha_0^a(k') \frac{\partial \tilde{V}_0^*(k', Y_0(k'))}{\partial h_0^a(k')} + \frac{(\sigma_0^{b,a}(k'))^2}{2} \frac{\partial^2 \tilde{V}_0^*(k, Y_0(k'))}{\partial h_0^a(k')^2} = 0 \end{aligned} \quad (17)$$

With terminal condition $\tilde{V}_0^*(0, Y_0(0)) = \epsilon(Q_0(0))$. And, as before, the optimal MF power strategy $\tilde{p}^*(k, Y_0(k))$ can then be derived from the optimal Bellman function $\tilde{V}_0^*$, as:

$$\tilde{p}^*(k, Y_0(k)) = \left[\frac{\partial \tilde{V}_0^*(k, Y_0(k))}{\partial Q_0(k)} \frac{\Delta_w}{\log(2)} - \frac{\sigma_n^2 + I_0(\pi_0(k))}{h_0^a(k)} \right]^+ \quad (18)$$

Note that in Section IV, we had to solve $|\bar{\mathcal{N}}|$ HJB equations, iteratively, in order to compute the optimal power strategy to be used by every user $i \in \bar{\mathcal{N}}$, in response to its current and estimated future interference I_i and parameters X_i. By considering a unique MF power strategy, we only solve a single HJB equation, which depends on the current parameters $Y_0(k)$ of any typical user $i = 0$ in the system, at RE $k \in K_0$. To this end, we reduce the mathematical complexity of the optimization analysis.

Hyp. 3 - Social Interaction as a Mean Field Interference: The social interaction between users, modeling the concurrent transmissions, consists of the interference $I_0(\pi_0(k'))$ perceived by our typical user, which was previously defined with (7). Instead, the MF interference I_0 now writes using the Mean Field power strategy $\tilde{p}$. Elements of stochastic geometry can also come in handy and can help define a more accurate Mean Field interference term I_0, as suggested in [15], [9]. To do so, in our problem, let us first introduce $M_0^{Y(k)}(k', Y(k'))$, the probability of our typical user to be in state $Y(k')$ at resource element $k' \in K_0, k' \leq k$, assuming its present state is $Y(k)$ at resource element $k \in K_0$. It can be demonstrated that $M_0^{Y(k)}(k', Y(k'))$ follows a unique Fokker-Planck-Kolmogorov (FPK) Equation, which writes $\forall k' \in K_0, k' \leq k$.

$$\begin{aligned} &\frac{\partial M_0^{Y(k)}(k', Y(k'))}{\partial k'} \frac{1}{\Delta_t} = -\alpha_0^a(k') \frac{\partial M_0^{Y(k)}(k', Y(k'))}{\partial h_0^a(k')} \\ &+ \frac{(\sigma_0^{b,a}(k'))^2}{2} \frac{\partial^2 M_0^{Y(k)}(k', Y(k'))}{\partial h_0^a(k')^2} \\ &+ \frac{\Delta_w}{\log(2)} M_0^{Y(k)}(k', Y(k')) \frac{\partial^2 \tilde{V}_0^*(k', Y(k'))}{\partial Q_0(k')^2} \frac{1}{\frac{\partial \tilde{V}_0^*(k', Y(k'))}{\partial Q_0(k')}} \\ &+ \log_{2+}\left(\frac{\partial \tilde{V}_0^*(k', Y(k'))}{\partial Q_0(k')} \frac{h_0^a(k')\Delta_w}{\log(2)(\sigma_n^2 + I_0(\pi_0(k')))} \right) \Delta_w \frac{\partial M_0^{Y(k)}(k', Y(k'))}{\partial Q_0(k')} \end{aligned} \quad (19)$$

With initial condition $\forall Y, M_0^{Y(k)}(k, Y) = \delta(Y = Y(k))$. As before, the proof for this equation can be found in [12]. It should however be noted that a FPK equation, modeling the global evolution of the system, as in [6], [7], [8] can not be defined in our problem, as we have to take into account the resource allocation performed initially, and the requests arrivals and departures. Still, the system can define, at the beginning of RE k, an estimate of the expected future interference $I_0(\pi_0(k'))$ for our typical user, at future RE $k' \in K_0, k' < k$, by reusing $M_0^{Y(k)}(k', Y(k'))$ and $\tilde{p}$. If we define the set of active users on resource element k' as $\Gamma(\pi_0(k')) = \left\{i' \in \bar{\mathcal{N}} \mid \exists l, \pi'_i(l) = \pi_0(k')\right\}$, the interference term, initially modeled as (7), can be rewritten as:

$$\begin{aligned} I_0(\pi_0(k')) &= |\Gamma(\pi_0(k'))| \mathbb{E}\left[h_0^{int} p_0^{int}\right] - \int_{Y_0(k')} h_0^a(k') \tilde{p}(k', Y_0(k')) M_0^{Y_0(k)}(k', Y_0(k')) dY_0(k') \\ &\overset{(a)}{=} |\Gamma(\pi_0(k'))| \mathbb{E}\left[h_0^{int}\right] \mathbb{E}\left[p_0^{int}\right] - \int_{Y_0(k')} h_0^a(k') \tilde{p}(k', Y_0(k')) dY_0(k') \end{aligned} \tag{20}$$

Where (a) follows from the decorrelation between the interference channels and the optimal powers, and:

$$\begin{aligned} \mathbb{E}\left[h_0^{int}\right] &= \frac{1}{|\Gamma(\pi_0(k'))|} \sum_{i' \in \Gamma(\pi_0(k'))} \mathbb{E}\left[h_{A'_i(\pi_0^t(k')),i}(\pi_0^t(k'))\right] \\ \mathbb{E}\left[p_0^{int}\right] &= \frac{1}{|\Gamma(\pi_0(k'))|} \sum_{i' \in \Gamma(\pi_0(k'))} \mathbb{E}\left[\tilde{p}(\pi_{i'}^{-1}(\pi_0(k'))), Y_{i'}(\pi_{i'}^{-1}(\pi_0(k')))\right] \end{aligned} \tag{21}$$

The first element $\mathbb{E}\left[h_0^{int}\right]$ represents the average expected interference channel between the BSs assigned to active users in set $\Gamma(\pi_0(k'))$ and our typical user, whereas $\mathbb{E}\left[p_0^{int}\right]$ consists of the average transmission powers used by active elements in set $\Gamma(\pi_0(k'))$. The first one can be expressed using elements of stochastic geometry, as in [15], [9]. The second one is expressed using the MF power strategy $\tilde{p}$ and the expected state probabilities M.

$$\begin{aligned} \mathbb{E}\left[h_0^{int}\right] &= \lambda\left(\pi + \frac{2\pi}{\beta-2}\left(1 - \frac{1}{\Delta_d^{(\beta-2)}}\right)\right) \\ \mathbb{E}\left[p_0^{int}\right] &= \sum_{i' \in \Gamma(\pi_0(k'))} M_{i'}^{Y_{i'}(\theta_{0,i'}^k)}(\theta_{0,i'}^{k'}, Y_{i'}(\theta_{0,i'}^{k'})) \tilde{p}(\theta_{0,i'}^{k'}, Y_{i'}(\theta_{0,i'}^{k'})) \end{aligned} \tag{22}$$

Where $\theta_{0,i'}^k = \pi_{i'}^{-1}(\pi_0(k))$. By doing so, we can then formulate an estimate of the expected future interference, for any typical user and any future RE. Note that $|\Gamma(\pi_0(k'))|$ is known exactly in advance, as we have assumed that the system has perfect knowledge of the request arrivals, future UE-BS assignments and future resource allocation. If such knowledge was not available, we could express $\mathbb{E}\left[|\Gamma(\pi_0(k'))|\right] = P_{act}\lambda\pi\Delta_d^2$, where P_{act} is the probability of any BS in the system to be active: according to [16], we have $P_{act} \approx 1 - (1 + \frac{\lambda_u}{3.5\lambda})^{(-3.5)}$. It can be demonstrated that $\mathbb{E}\left[p_0^{int}\right]$ is bounded, and thus, the expected future interference I_0, remains also bounded, when Δ_d grows large.

VI. Numerical simulations and performance comparison

A. Numerical Method for Mean Field Optimum Computation

In order to compute the optimal power strategies $p_i^*(k)$ to be used by any user $i \in \bar{\mathcal{N}}$ at the beginning of any RE $k \in K_i$, we consider an iterative process, which goes as follows:

- Compute the unique MF power strategy $\tilde{p}$ to be used by any typical user $i = 0$, on any RE from its assigned set of REs K_0, by solving the MFG HJB (17) and by using (18).
- Compute the expected evolution of any typical user $i = 0$, if every user implements the unique MF power strategy $\tilde{p}$, by solving the FPK equations (19) and using (18).
- Update the expected future interference term using the unique MF power strategy $\tilde{p}$ and the expected trajectories for all users $(M_i)_{i \in \bar{\mathcal{N}}}$, thanks to (20).

TABLE I
Simulations Parameters.

Region size Δ_d	5000m
Time slot (TS) duration Δ_t	0.5ms
Number of TSs T	500
Bandwidth elements W	6
Bandwidth unitary length Δ_w	5 MHz
Path loss exponent β	3
Noise power σ_n	-104 dBm
BS spatial density λ	$1.6e^{-5}$ BS/m^2(400 BS on avg.)
Average inter-arrival times μ_{at}	0.15ms (100 UEs active on each TS, on avg.)
Average service times μ_{st}	15ms (30 TSs on avg.)
Average data request μ_{dr}	500 kb

The process is repeated until a convergence on $\tilde{p}$ is observed, which indicates the MFG equilibrium has been reached. The optimal power strategy, for user i is then obtained as $\forall i, \forall k, p_i(k) = \tilde{p}(k, Y_i(k))$. In that sense, the process resembles to the one considered for the multi-user DSG in Section IV, but it is known to have a faster convergence [17], [7]. It should however be noted, that solving the HJB equation (17) analytically is impossible, and we will have to solve it numerically. Browsing through all the possible scenarios for $(\alpha_i^a(k'))_{\substack{i \in \bar{\mathcal{N}} \\ k' \in K_i}}$ and $(\sigma_i^{b,a}(k'))_{\substack{i \in \bar{\mathcal{N}} \\ k' \in K_i}}$, might lead to numerous sub-HJB PDEs, with a computational cost, higher than the one required in the multi-user DSG from Section IV. For simplicity, we will then assume in the simulations that all users $i \in \bar{\mathcal{N}}$ belong to the same mobility class and have similar channel dynamics, i.e. $\forall i \in \bar{\mathcal{N}}, \forall k \in K_i$, $\alpha_i^a(k) = \alpha^a(k)$ and $\sigma_i^{b,a}(k) = \sigma^{b,a}(k)$.

B. Performance Index and Simulation Parameters

In this section, we compare the performance of the MFG scheduler, to a reference scheduler, namely the equal-bit one, introduced in [10]. This scheduler, simply transmits at a constant rate, no matter what the channel gain and interference is. In that sense, it is able to exploit the offered latency, but is unable to take into account any available knowledge about the channel state evolution, and the future requests arrivals/departures. For both schedulers, the performance index we consider for each user i consists of the energy efficiency E_i^{eff} (in bits/J), defined as the ratio between the transmitted data D_i^r and the total consumed energy $E_i^*(\mathcal{P}_i^*)$. To highlight potential performance gains, we run numerical simulations, with standard parameters, listed in Table I hereafter. Also, we have assumed the deterministic mobility α^a is such that the channel gain of every user, oscillates around its initial value, computed when the request arrives, with $+/-20dB$. Also, the channel evolutions are affected by Brownian motions with variances $\sigma^{b,a} = 2000\Delta_t$. We have represented in Figure 1, a realization of the channel evolution for a random user in the network.

In Figure 2, we have represented the energy efficiencies of the users $i \in \bar{\mathcal{N}}$, for two different scheduling strategies: the MFG-based scheduling and the equal-bit scheduler. It appears that the proposed MFG scheduler is able to take into account the channel dynamics, to adapt the optimal transmission to be used at each RE assigned to every user, in a more efficient manner than the state-of-the-art equal-bit scheduler, used for comparison. This leads to an energy efficiency gain of a factor 5.4 on average. It should however be noted that the gain depends on two factors [12], [18]. First, the channel variations α^a affect the performance gain observed here, in particular, if the channel was not varying and no stochasticity was considered ($\sigma^{b,a} = 0$), then the equal-bit scheduler would be the optimal scheduler. As a consequence, a performance gain is only observed if the users mobility is such that we observe variations in the transmission channels, wrt. time. Secondly, the channel evolution prediction formulated in (8), must be accurate enough, as the uncertainty $\sigma^{b,a}$ can rapidly degrade the performance of the MFG scheduler, and can lead to scenarios where the MFG scheduler will find the future so uncertain, that it will attempt to transmit as soon as possible. This can rapidly degrade the performance of the MFG scheduler, as it would become unable to exploit the offered latency appropriately.

VII. Conclusion

In this paper, we have characterized the optimal energy-efficient scheduling strategies in UDNs, by taking into account both the spatial and temporal aspects related to network deployment and users requests. The conducted analysis demonstrates how Mean Field Games can be coupled with elements of Stochastic Geometry and Queueing Theory, to reduce the inherent mathematical complexity of the optimization. Through the provided numerical simulations, we highlight the notable performance gains offered by a MFG-based scheduling approach, in terms of average energy efficiency, compared to an equal-bit scheduler. Future work on this topic will further investigate how to relax the two core hypotheses we made during the analysis, namely the perfect a priori future knowledge about requests and UE-BS assignments, as well as the identical mobility pattern of users. Future work will also investigate other reference schedulers, as well as possible heuristics approximating the MFG strategies.

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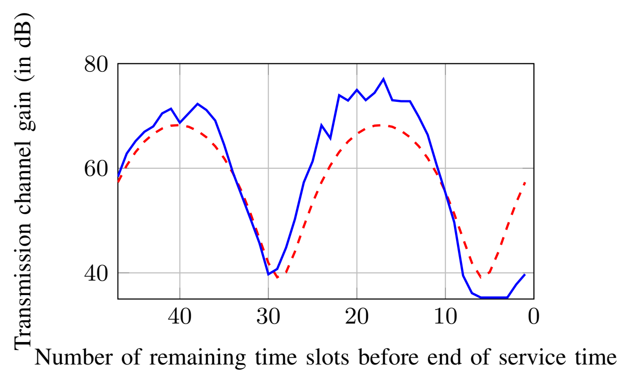

Fig. 1. Channel Evolution of a random user. In dashed red, the deterministic prediction (with no stochasticity), in blue the effective channel realization.

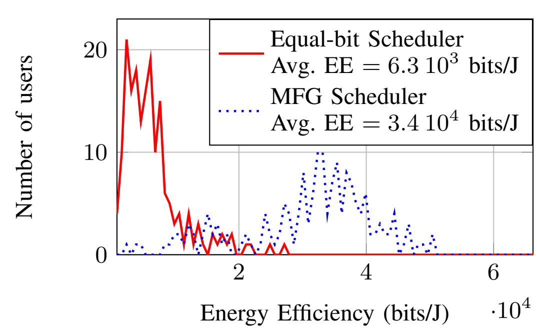

Fig. 2. Histogram plot - Energy Efficiency (bits/J) distribution among the users in the network, for the two considered scheduling strategies